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A model of ideological struggle

Nikolay K. Vitanov,[1,*] Zlatinka I. Dimitrova,[2] and Marcel Ausloos[3,†]

[1]*Institute of Mechanics, Bulgarian Academy of Sciences, Akad. G. Bonchev Str., Bl. 4, 1113 Sofia, Bulgaria*

[2]*Institute of Solid State Physics, Bulgarian Academy of Sciences, Blvd. Tzarigradsko Chaussee 72, 1784, Sofia, Bulgaria*

[3]*GRAPES, B5 Sart-Tilman, B-4000 Liège, Euroland*

A general model for opinion formation and competition, like in ideological struggles is formulated. The underlying set is a closed one, like a country but in which the population size is variable in time. Several ideologies compete to increase their number of adepts. Such followers can be either converted from one ideology to another or become followers of an ideology though being previously ideologically-free. A reverse process is also allowed. We consider two kinds of conversion: unitary conversion, e.g. by means of mass communication tools, or binary conversion, e.g. by means of interactions between people. It is found that the steady state,when it exists, depends on the number of ideologies. Moreover when the number of ideologies increases some tension arises between them. This tension can change in the course of time. We propose to measure the ideology tensions through an appropriately defined scale index.

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I. INTRODUCTION

A. Nonlinear systems, opinion formation and population dynamics

The case of opinion formation [1] - [5] in sociology is seen in recent physics works as analogous to state phase evolution in non-equilibrium systems. Applications to population dynamics [6], extinction of populations [7], animal and human migration [8], policy and politics [9], languages [10], religions [11] - [14] are found to be similar to epidemics [15], forest fires [16] and other self-organizing systems much studied in statistical physics. No general pattern is however available and situations can be very varied. Whence it seems appropriate to continue considering the questions through the methods used in the theory of turbulence [17] - [20], low-dimensional dynamical systems [21] - [23] or theory of nonlinear waves [24] - [26] but *exordium* to go back to the classical Verhulst ideas and Lotka-Volterra model by introducing some realistic conditions on the growth ratios and on the interaction coefficients between the populations [27] - [31]. Indeed there is then a connexion to the problem of extinction of populations [7], religions [11], languages [32], and to the very modern question of internet governance in which the *old* stakeholders, i.e. the most powerful actors, and a variable set of *new* participants are somewhat abused or lacking cohesion in their reaction [33].

Interestingly this demands at some stage a consideration of the connexion between economical and social physics and social dynamics, including the analysis of time series [34] - [39] though with some caveat due to either size or debatable data value. Whence the need for a theoretical approach and some modelization in order to focus any data gathering toward useful input in further work.

B. Organization of the paper

In Sect. 2, we formulate a general/mathematical model in a finite system size, *we emphasize,* allowing for a changing population size. In a realistic way we consider people unaffected by the available ideologies as well as conversions to ideologies. Two mechanisms are discussed by which the followers of an ideology can increase: *unitary* conversion (a citizen is converted by means of forms of mass communication such as newspapers, radio or television channels) and *binary* conversion (a citizen is converted by interpersonal contacts with other citizens).

*Electronic address: vitanov@imbm.bas.bg
†Electronic address: marcel.ausloos@ulg.ac.be

In Sect. 3 we study the case when only one ideology spreads among the population. The whole country population is found to evolve towards an equilibrium state. In this state some part of the people becomes followers of the ideology and the remaining ones are not followers of the ideology. The fraction of adepts depends on the intensity of the unitary and binary conversion as well as on the ability of the ideology to reduce the dissatisfaction among its followers. In Sect. 4, the case of two ideologies is examined. The introduction of a second ideology leads to tensions between the ideologies as the number of followers drops in comparison to the case when each of the ideologies is alone in the country. The tension can be quantified on a scale that can be considered to be an index of ideological tensions in the society.

In Sect. 5, in the case of three ideologies, we briefly show that a chaotic behavior is seen to occur among many other solutions. This seems close to intuition. In Sect. 6, conclusions are outlined and connected for relevance sake to a usual observation that the coexistence of ideologies implies the existence of tensions between them.

II. MATHEMATICAL FORMULATION OF THE MODEL

Let us consider a set (country) with a population of N agents. We are going to consider that the population is divided into $n+1$ factions: n factions each with a different specific ideology, such that the number of members in the corresponding populations are $N_1, N_2, \ldots, N_n$, and a fraction N_0 of people which are not followers of any ideology at a given moment of time. Then

$$N = N_0 + \sum_{i=1}^{n} N_i \tag{1}$$

We assume that the overall population evolves according to the generalized Verhulst law

$$\begin{aligned}\frac{dN}{dt} &= r(t, N, N_1, \ldots, N_n, p_\mu) N \times \\ &\left[1 - \frac{N}{C(t, N, N_1, \ldots, N_n, p_\mu)}\right]\end{aligned} \tag{2}$$

where $p_\mu = (p_1, \ldots, p_m)$ are parameters, describing the environment.

The growth process is constantly disrupted by small extinction events, as in [40], monitored through $r(t, N, N_1, \ldots, N_n, p_\mu)$. r is the overall population growth rate; r can be positive or negative. In this paper we shall consider the case $r > 0$, i.e. we shall study the spreading and competition between ideologies in a country with a growing total population. $C(t, N, N_1, \ldots, N_n, p_\mu)$ is the maximum possible population of the country (its so called carrying capacity). In every ideological population i we have to account for the following processes: deaths, dissatisfaction, unitary conversion, and binary conversion.

1. First, we expect a decrease of the number of followers of an ideology through death or dissatisfaction with the ideology, i.e. through a term $r_i N_i$, where $r_i \leq 0$. In general $r_i = r_i(t, N, N_1, \ldots, N_n, p_\mu)$.

2. Unitary conversion: such a conversion from one ideology to another is made without direct contact between the followers of different ideologies. The conversion happens through the information environment of the population. Elements of this environment for example are the newspapers, the radio stations, television channels, printed propaganda materials or mass events such as speeches during election campaigns. Excluded are the direct interpersonal contacts which lead to the binary conversion described below. In order to model the unitary conversion we assume that the number of people converted from ideology j to ideology i is proportional to the number N_j of the followers of the ideology j. An f_{ij} coefficient characterizes the intensity with which this conversion occurs. The corresponding modeling term is $f_{ij} N_j$. We assume that $f_{ii} = 0$. In addition a term $f_{i0} N_0$ describes the unitary conversion toward ideology i from the N_0 people who were not followers of any ideology at the corresponding moment of time. In general

$$f_{ij} = f_{ij}(t, N, N_1, \ldots, N_n, p_\mu, C)$$

$$f_{i0} = f_{i0}(t, N, N_1, \ldots, N_n, p_\mu, C)$$

3. Binary conversion: $b_{ijk} N_j N_k$. In general this term describes the conversion to the i-th ideology because of direct interaction between members of the j-th and k-th ideology. We assume that the intensity of the interpersonal

contacts is proportional to the numbers N_j and N_k of the followers of the two ideologies. The coefficient characterizing the intensity of the binary conversion is b_{ijk}. The larger is b_{ijk}, the more people are converted to the i-th ideology. In general the binary conversion coefficients can be $b_{ijk} = b_{ijk}(t, N, N_1, \dots, N_n, p_\mu, C)$. Of course $b_{iii} = 0$: there is no self-conversion.

The equation for the evolution of the followers of the ideology i becomes

$$\begin{aligned}\frac{dN_i}{dt} &= r_i(t, N, N_1, \dots, N_n, p_\mu, C)N_i + \\ & f_{i0}(t, N, N_1, \dots, N_n, p_\mu, C)N_0 + \\ & \sum_{j=1}^{n} f_{ij}(t, N, N_1, \dots, N_n, p_\mu, C)N_j + \\ & b_{i0}(t, N, N_1, \dots, N_n, p_\mu, C)N_iN_0 + \\ & \sum_{j=1}^{n}\sum_{k=1}^{n} b_{ijk}(t, N, N_1, \dots, N_n, p_\mu, C)N_jN_k \end{aligned} \tag{3}$$

In general we can have a co-evolution of the environment and the populations, i.e. $p_\mu = p_\mu(N, N_1, \dots, N_n, C, t)$, but this will not be discussed here.

Indeed in this paper we shall discuss the simplest version of the model namely the case in which all the coefficients are time and p_μ independent. Then the model system becomes geared by

$$N = N_0 + \sum_{i=1}^{n} N_i \tag{4}$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{C}\right) \tag{5}$$

$$\frac{dN_i}{dt} = r_iN_i + f_{i0}N_0 + \sum_{j=1}^{n} f_{ij}N_j + b_{i0}N_iN_0 + \sum_{j=1}^{n}\sum_{k=1}^{n} b_{ijk}N_jN_k \tag{6}$$

Two remarks are in order.

1. Notice that arbitrary values are not allowed for the coefficients of the model. They must have values such that N, N_0, N_1, …, N_n be nonnegative at each t.

2. Let us consider the i-th population and the binary conversion characterized by the coefficients b_{ijk} and b_{ikj} where j and k are different from i. One could at first think that $b_{ijk}N_jN_k$ and $b_{ikj}N_kN_j$ describe one and the same process in which the interaction between followers of the j-th and k-th ideology leads to a conversion of these to the i-th ideology. In general however one should not identify the two terms. In so doing in the general model we retain one additional degree of freedom, i.e., that which allows to distinguish between the ideology that is of the initiator of the interaction and the ideology of someone who is apparently simply a participant in the interaction.

Below we shall consider the dynamics of populations of followers of the ideologies for the cases of presence of 1,2 or 3 ideologies in the country.

III. THE CASE OF ONE IDEOLOGY

In the case of spreading of one ideology the population of the country is divided into two groups: N_1 followers of the single ideology and N_0 people who are not followers of this ideology at the corresponding time. Let us first discuss the case when only the unitary conversion scheme exists, as possibly moving the ideology-free population toward the single ideology, i.e. f_{10} is finite. Let the initial conditions be $N(t=0) = N(0)$ and $N_1(t=0) = N_1(0)$. The solution of the model system is

$$N(t) = \frac{CN(0)}{N(0) + (C - N(0))e^{-rt}} \tag{7}$$

$$N_1(t) = e^{-(f_{10}-r_1)t}\Bigg\{N_1(0) + \frac{cf_{10}}{r} \times$$
$$\Bigg[\Phi\Bigg(-\frac{C-N(0)}{N(0)}, 1, -\frac{f_{10}-r_1}{r}\Bigg) -$$
$$e^{t(f_{10}-r_1)}\Phi\Bigg(-\frac{C-N(0)}{N(0)e^{rt}}, 1, -\frac{f_{10}-r_1}{r}\Bigg)\Bigg]\Bigg\} \tag{8}$$

$$N_0(t) = N(t) - N_1(t) \tag{9}$$

where Φ is the special function

$$\Phi(z,a,v) = \sum_{n=0}^{\infty} \frac{z^n}{(v+n)^a} \; ; \qquad \mid z \mid < 1 \tag{10}$$

The condition $\mid z \mid < 1$ is equivalent to $N^* > C/2$. This means that the solution (7) describes the development in the case of a densely populated territory. For $N^* \leq C/2$, Φ can tend to ∞ and we can only obtain a numerical solution of the model system of equations.

The obtained solution describes an evolution in which the total population N reaches asymptotically the carrying capacity C of the environment. The number of adepts of the ideology reaches an equilibrium value which corresponds to the fixed point of the model equation for $\frac{dN_1}{dt}$. This fixed point is

$$\hat{N}_1 = \frac{Cf_{10}}{f_{10}-r_1}$$

The number of people which are not followers of the ideology asymptotically tends to $N_0 = C - \hat{N}_1$. As a numerical example let $C = 1$, $f_{10} = 0.03$ and $r_1 = -0.02$, then $\hat{N}_1 = 0.6$ which means that the evolution of the system leads to an asymptotic state in which 60 % of the population are followers of the ideology and 40 % are not.

Now let not only unitary but also binary conversion processes be possible. The evolution in this case cannot be investigated analytically. However the asymptotic behavior for N_1 can be obtained when the total population N has reached the carrying capacity C of the environment. For this asymptotic state the evolution of N_1 reads

$$\frac{dN_1}{dt} = r_1 N_1 + f_{10}(C - N_1) + b_{10}N_1(C - N_1) \tag{11}$$

There exist two fixed points but only one of them satisfies the requirement $\hat{N}_1 > 0$. This fixed point is

$$\hat{N}_1 = \frac{(r_1 - f_{10} + b_{10}C) + \sqrt{(r_1 - f_{10} + b_{10}C)^2 + 4b_{10}f_{10}C}}{2b_{10}} \tag{12}$$

The equation (11) has an analytical solution. The solution depends on whether $N_1 > \hat{N}_1$ or $N_1 < \hat{N}_1$. The two cases can be realized respectively when $N_1(0) > \hat{N}_1$ and $N_1(0) < \hat{N}_1$. If $N_1(0) > \hat{N}_1$ then

$$N_1 = \frac{X_1}{Y_1} \tag{13}$$

where

$$X_1 = r_1 - f_{10} + b_{10}C + \sqrt{(r_1 - f_{10} + b_{10}C)^2 + 4b_{10}f_{10}C} +$$
$$e^{-(t+\tau)(r_1 - f_{10} + b_{10}C)} \times$$
$$\Bigg(r_1 - f_{10} + b_{10}C - \sqrt{(r_1 - f_{10} + b_{10}C)^2 + 4b_{10}f_{10}C}\Bigg)$$

$$Y_1 = 2b_{10}\Bigg(1 - e^{-(t+\tau)}(r_1 - f_{10} + b_{10}C)\Bigg)$$

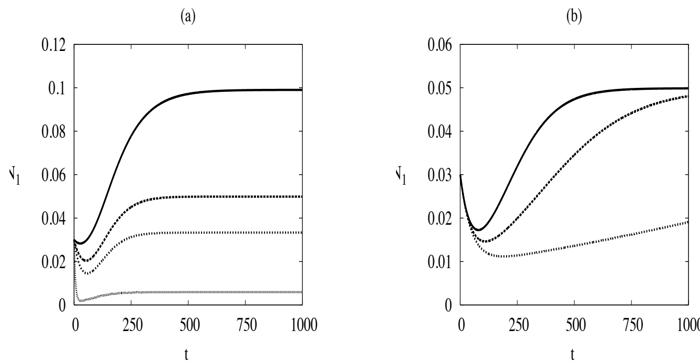

FIG. 1: (a): Illustration of the inertial growth and its dependence on the parameter r_1. The other parameters are: $C = 1$, $r = 0.2$, $f_{10} = b_{10} = 0.001$. (b): Illustration of the inertial growth and its dependence on the parameter r, with $C = 1$, $r_1 = -0.02$, $f_{10} = b_{10} = 0.01$

and where the characteristic time τ is here given by

$$\tau = -\frac{1}{r_1 - f_{10} + b_{10}C} \times \ln\left(\frac{2b_{10}N_1(0) + Z}{2b_{10}N_1(0) + r_1 - f_{10} + b_{10}C - Z}\right) \tag{14}$$

where $Z = \sqrt{(r_1 - f_{10} + b_{10}C)^2 + 4b_{10}f_{10}C}$.

For the case $0 < N_1(0) < \hat{N}_1$

$$N_1 = \frac{X_2}{Y_2} \tag{15}$$

where

$$X_2 = X_1$$

$$Y_2 = 2b_{10}\left(1 + e^{-(t+\tau)}(r_1 - f_{10} + b_{10}C)\right)$$

Let us now discuss the time behavior of the number of followers of the ideology. There are three possibilities

1. $\frac{dN_1}{dt} > 0$ for all t, i.e. the number of followers increases monotonically. For the particular case of only a unitary conversion process the condition reads $-\frac{r_1}{f_{10}} < \frac{N_0}{N_1}$

2. $\frac{dN_2}{dt} < 0$ for all t, i.e. the number of followers decreases monotonically. For the case of only unitary conversions, the condition reads $-\frac{r_1}{f_{10}} > \frac{N_0}{N_1}$

3. The most interesting case is when $\frac{dN_1}{dt}$ can change sign with increasing t. The following effect can be observed: the number of followers of the ideology can increase despite the fact that $r_1 < 0$. The reason for this effect is the increasing number N_0, occurring because of the fast enough growth of the population of the country. When N_0 is small the term containing r_1 dominates and N_1 decreases. But in the course of time N_0 increases. Then the conversion begins to dominate over dissatisfaction and the number of the followers of the ideology begins to increase. We shall call such a kind of growth of the followers of the ideology an inertial growth.

Fig. 1 illustrates an inertial growth. In Fig. 1a, it can be observed that the process of initial shrinking and then of inertial growth can exist for a large range of coefficient values. Inertial growth can exist even if the ideology is weak with respect to its keeping of followers, i.e. when r_1 has large negative values. Note that the figure illustrates the case when the carrying capacity of the environment is a constant. If the carrying capacity would change, one could observe sequences of phases of inertial growth and shrinking. Fig. 1b shows that a small growth rate r, i.e. of the total population, leads to a slowing down of the inertial growth process.

IV. CASE OF TWO IDEOLOGIES: THE IDEOLOGICAL TENSION

In this section we will discuss the competition for adepts that the presence of a second ideology introduces; this is leading to a measurable conflictual tension, as will be shown.

Let us consider the model system for the case of two ideologies with populations of followers N_1 and N_2; we assume that all parameters are kept constant. We have

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{C}\right) \tag{16}$$

$$\frac{dN_1}{dt} = r_1 N_1 + f_{10} N_0 + f_{12} N_2 + b_{10} N_0 N_1 + (b_{112} + b_{121}) N_1 N_2 + b_{122} N_2^2 \tag{17}$$

$$\frac{dN_2}{dt} = r_2 N_2 + f_{20} N_0 + f_{21} N_1 + b_{20} N_0 N_2 + b_{211} N_1^2 + (b_{212} + b_{221}) N_1 N_2 \tag{18}$$

$$N = N_0 + N_1 + N_2 \tag{19}$$

Let us quantify the tension between ideologies by means of an asymptotic analysis. We discuss the case of only unitary conversion of members of the population that are not followers of any of the ideologies. In order to emphasize the unitary conversion effects let us assume that the binary conversion as well as the unitary conversion from one ideology to the the other one are negligible. In this case $f_{12} = f_{21} = 0$ and $b_{112} = b_{122} = b_{121} = b_{211} = b_{212} = b_{221} = 0$. In addition let us consider the asymptotic case in which the total population has reached the carrying capacity : $N = C$. The equilibrium state is characterized by the fixed points :

$$\breve{N}_1 = \frac{C r_2 f_{10}}{r_1 f_{20} + f_1 r_2 - r_1 r_2}, \quad \breve{N}_2 = \frac{C r_1 f_{20}}{r_1 f_{20} + f_1 r_2 - r_1 r_2} \tag{20}$$

If the ideologies were without competition their size would be; see the previous section

$$\hat{N}_1 = \frac{C f_{10}}{f_{10} - r_1}; \quad \hat{N}_2 = \frac{C f_{20}}{f_{20} - r_2} \tag{21}$$

It can thus be observed that the popularity of an ideology shrinks when competing ideology or ideologies spread around the country. Let us evaluate this shrinking. We have

$$\frac{\breve{N}_1}{\hat{N}_1} = \frac{1}{1 + \frac{r_1 f_{20}}{r_2 (f_{10} - r_1)}}, \quad \frac{\breve{N}_2}{\hat{N}_2} = \frac{1}{1 + \frac{r_2 f_{10}}{r_1 (f_{20} - r_2)}} \tag{22}$$

As a numerical example let $r_1 = r_2 = -0.01$ and $f_{10} = f_{20} = 0.02$. Then $\breve{N}_1/\hat{N}_1 = 0.6$, i.e. the number of followers of the ideology 1 descreases by 40%. Of course this causes some tension between the ideologies. A measure of this tension can be through the index

$$T_{i;k} = 1 - \frac{N_i^{(k)}}{\hat{N}_i}, \tag{23}$$

where $N_i^{(k)}$ is the population of the followers of the i-th ideology when the k-th ideology is presented in the country too. If the ideology is alone then $N_1^{(1)} = \hat{N}_1$ and the tension index is $T_{1;1} = 0$. If N_1 decreases because of the competition with the second ideology, then the tension between the ideologies characterised by the tension index $T_{i;k}$ increases. The above definition for the tension holds even if N_1 follows some time dependent trajectory.

The tension index can be generalized for the case of an arbitrary number of ideologies in the country (next section for example). Let m ideologies be presented in the country. The tension on i-th ideology in presence of two other ideologies, k and l is

$$T_{i;k,l}(t) = 1 - \frac{N_i^{(k,l)}(t)}{\hat{N}_i} \tag{24}$$

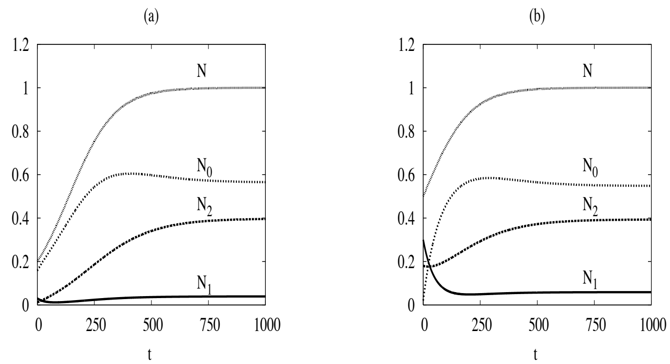

FIG. 2: Evolution of the total population N, number of followers of the ideologies, N_1 and N_2 and the number of N_0 of people that are not followers of any ideology. (a): $r_1 = -0.03$. Initial conditions: $N_1(0) = 0.03$, $N_2(0) = 0.012$, $N(0) = 0.2$; (b): $r_1 = -0.02$. Initial conditions: $N_1(0) = 0.3$, $N_2(0) = 0.18$, $N(0) = 0.5$. In both figures : $r_2 = -0.005$, $r_3 = 0.01$, $C = 1$, $f_{10} = 0.001$, $f_{20} = 0.003$, $f_{11} = f_{22} = 0$, $f_{12} = f_{21} = 0.001$, $b_{10} = b_{20} = 0.001$, $b_{111} = b_{222} = 0$; $b_{112} = b_{121} = b_{122} = b_{211} = b_{212} = b_{221} = 0.001$.

where $N_i^{(k,l)}$ is the population of followers of the i-th ideology when the ideologies k and l operate in the country too. The tension on i-th ideology in presence of three other ideologies, j, k, l is

$$T_{i;j,k,l}(t) = 1 - \frac{N_i^{(j,k,l)}(t)}{\hat{N}_i} \tag{25}$$

where $N_i^{(j,k,l)}$ is the number of the followers of the i-th ideology in presence of ideologies j, k, l. In such a way we can define a series of indices for the quantification of the tensions among ideologies competing for followers (in the same country).

Fig. 2 shows typical results from the numerical investigation of (16) -(19). Fig. 2a shows the purely inertial growth of a population of followers of ideology 2 and its decline, followed by the inertial growth of the population of followers of ideology 1. In Fig. 2b one can observe an initial decline followed by an inertial growth of the number of followers of *both* ideologies. These results can be usefully compared to a model of telecommunication competition [45] where it is concluded that (we quote) "schemes targeting local cliques within the network are more successful at gaining a larger share of the population than those that target users randomly at a global scale (e.g., television commercials, print ads, etc.). This suggests that success in the competition is dependent not only on the number of individuals in the population but also on how they are connected in the network. The network in our above investigation is such that all agents are fully connected with each other; we consider a fully connected graph. This is equivalent to a mean field approximation study. Notice that the links are weighted through the f_{ij} and b_{ijk} coefficients.

V. CASE OF THREE IDEOLOGIES

The case of three ideologies will not be treated in full here. It can be easily understood that the above procedure can be extended to the case of multiple ideologies. We only restrict our presentation to the main points and illustrate the newness. In the case of three ideologies the model system (4) - (6) has an analytical solution in the following conditions. First, let the binary conversion be negligible: $b_{ijk} = 0$, $b_{i0} = 0$. Then the model system reduces to

$$N = N_0 + \sum_{i=1}^{n} N_1 \tag{26}$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{C}\right) \tag{27}$$

$$\frac{dN_i}{dt} = (r_i - f_{i0})N_i + f_{i0}N + \sum_{j=1;j\neq i}^{n} (f_{ij} - f_{i0})N_j \tag{28}$$

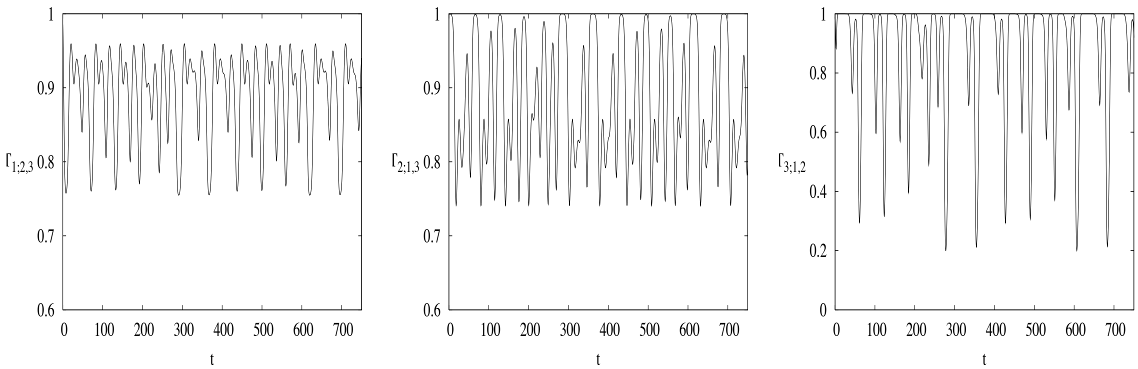

FIG. 3: An example of chaotic behavior of the tension indices for the case of three competing ideologies. The model system is as in Eq.(32). For the attractor shown the parameters are as follows: $\kappa_{11} = \kappa_1$,$\kappa_{12} = \kappa_1$, $\kappa_{13} = \kappa_2$, $\kappa_{21} = -\kappa_1$, $\kappa_{22} = -\kappa_2$, $\kappa_{23} = \kappa_2$, $\kappa_{31} = \kappa_3$, $\kappa_{32} = \kappa_2$, $\kappa_{33} = \kappa_2$. $C = 10$, $\kappa_1 = 0.5$, $\kappa_2 = 0.1$, $\kappa_3 = 1.63$, $b_{10} = 0.001$, $b_{20} = 0.002$, $b_{30} = 0.001$, $r_1 = -0.01$,$r_2 = -0.03$,$r_3 = -0.01$.

Let the initial conditions be $N(t=0) = N(0)$, $N_i(t=0) = N_i(0)$ and $N(0) > C/2$. Then the solution is

$$N(t) = \frac{CN(0)}{N(0) + (C - N(0))e^{-rt}} \tag{29}$$

$$\begin{aligned} N_i(t) = e^{-(f_{i0}-r_1)t}\Bigg\{ N_1(0) + \\ \frac{cf_{i0}}{r}\Bigg[\Phi\Bigg(-\frac{C-N(0)}{N(0)}, 1, -\frac{f_{i0}-r_i}{r}\Bigg) - \\ e^{t(f_{i0}-r_i)}\Phi\Bigg(-\frac{C-N(0)}{N(0)e^{rt}}, 1, -\frac{f_{i0}-r_i}{r}\Bigg)\Bigg]\Bigg\} \end{aligned} \tag{30}$$

$$N_0(t) = N(t) - \sum_{i=1}^{n} N_i(t) \tag{31}$$

where Φ is the special function defined in section 3. Above $i = 1, 2, \ldots, n$. When $n = 1$ one returns to the case of section 3. For $n = 2$ one recovers the solution of the case of section 4.

Chaotic change of the ideological tensions becomes possible when the number of the ideologies becomes equal to 3 or larger. This can be demonstrated easily as follows. Let the total population having reached the carrying capacity of the environment $N = C$. Let $f_{12} = f_{13} = f_{10} = 0$ and $b_{1jk} = b_{2jk} = b_{3jk} = 0$ for $j, k = 1, 2, 3$. Let also $f_{21} = f_{23} = f_{20} = 0$ and $f_{31} = f_{32} = f_{30} = 0$. Let us rewrite the parameters in the following way

$$r_1 + f_{11} = \kappa_{11} + \kappa_{12} + \kappa_{13}, b_{111} - b_{10} = -\kappa_{11}$$

$$b_{112} - b_{10} = -\kappa_{12}, b_{113} - b_{10} = -\kappa_{13}$$

$$r_2 + f_{22} = \kappa_{21} + \kappa_{22} + \kappa_{23}, b_{222} - b_{20} = -\kappa_{22}$$

$$b_{221} - b_{20} = -\kappa_{21}, b_{223} - b_{30} = -\kappa_{23}$$

$$r_3 + f_{33} = \kappa_{31} + \kappa_{32} + \kappa_{33}, b_{331} - b_{30} = -\kappa_{31}$$

$$b_{332} - b_{30} = -\kappa_{32}, b_{333} - b_{30} = -\kappa_{33}$$

The model system of equations becomes

$$\frac{dN_i}{dt} = N_i \sum_{j=1}^{3} \kappa_{ij}(1 - N_j) + b_{i0} C N_i; \qquad i, j = 1, 2, 3. \tag{32}$$

The existence of chaos in a particular case of (32) was discussed in [46]. Fig. 3 is an illustration of a case of chaotic change of the ideological tensions for the (32) corresponding model. $T_{1;2,3}$ is always significantly different from 0. This means that because of the competition with the ideologies 2 and 3 the ideology 1 remains at a significant "distance" from its most favorable state, - this one which would exist if which there was no competitor. In addition the ideology 1 copes relatively good with the situation as its tension index remains relatively distant from 1. The other two ideologies experience large tensions and from time to time are close to extinction. The ideology 2 evolves better than the ideology 3 which experiences large oscillations of the number of followers as consequence of the competition with the other ideologies. This illustration for a given set of parameters indicates the interest of the approach, since a few measures would allow to calibrate the parameters, in specific situations, whence would lead to considerations pertaining to forecasting science.

VI. CONCLUDING REMARKS

In this paper a general model for ideological competition was formulated. The model applies to cases in which countries have a variable total population which evolves according to the generalized Verhulst model. The discussion in the paper is concentrated on the cases of constant coefficients and when the total population of the country increases. An original ingredient concerns also the number of followers of an ideology which can increase without interpersonal contacts, but solely on the basis of so called unitary conversion, e.g. as a result of different forms of mass communication. The number of followers can also increase by means of binary conversion as a result of interpersonal contacts. It is emphasized that the conversion can be outside the competing ideologies of interacting agents.

The dynamics of the populations of followers of the ideologies is discussed for the case of one ideology and for the case of two and three competing ideologies, in Sect. 3-5, respectively. For the case of one ideology, the simple version of the general model describes the evolution to an equilibrium state in which the population consists of some amount of followers of the ideology and persons indifferent to the ideology. The introduction of a second ideology leads to some tension between the ideologies as the numbers of followers drop in comparison to the case when each of the ideologies is alone in the country. The ideological tensions can be quantified by a set of indices. A nonzero index is a characteristic feature of the competition. Each ideology most of the time, if not always, tries to set its indices of tension to 0, i.e. it tries to reach its maximum number of followers (which is the case when the ideology is alone in the country). This can be done by decreasing the number of followers of the other ideology on the territory.

We have indicated that chaos can exist (can be found) when the number of available ideologies increases above 2. The number of parameters is considerable, as in many realistic population evolution studies. However the set of parameters appears to be realistic enough to be calibrated in specific situations. This would lead to forecasting considerations. This and the other obtained results hint to good perspectives for applications of the methods of statistical physics, theory of networks, sociophysics, etc [41, 42, 43] to the problems of ideological competition. This will be a subject of future research.

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